IMPLEMENTATION OF SIGNAL PROCESSING OPERATIONS BY TRANSFORMS WITH RANDOM COEFFICIENTS FOR NEURONAL SYSTEMS MODELLING

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Abstract. This paper investigates signal processing networks in which randomness is an inherent feature like in biological neuronal networks. Signal processing operations are usually performed with algorithms requiring high-precision and order. It is thus interesting to investigate how signal processing operations could be realized in systems with inherent randomness which is apparent in neuronal networks. We are studying possible implementation of convolution and correlation operations based on generalized transform approach with rectangular matrices generated by random sequences. Conditions are formulated and illustrated how correlation and convolution operators can be computed with such matrices. We show next that increasing the size of matrices allows to decrease the precision of operations and to introduce substantial quantization and thresholding. The use of random matrices provides also for strong robustness to noise resulting from unreliable operation. We show also that the nonlinearity due to the quantization and thresholding leads naturally to the decorrelation of transformation vectors which might be useful for associative storage.

INTRODUCTION

Biological neuronal networks have immense complexity and give impression of having high degree of randomness in structure and operation. Despite the long period of research the principles of operation of these networks are still unclear. One of the fundamental problems is how specific algorithms are

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implemented by these networks taking their stark difference to known computational structures. There are several well-known computational models for artificial neural networks which are based on principles of adaptation in various forms [1]. An interesting problem is whether neural networks can be structured for explicit calculation of algorithms despite their apparent randomness. In this paper we are studying possibilities for the implementation of basic signal processing algorithms of convolution and correlation in networks with inherent randomness reminiscent of impression given by neuronal networks [2]. The problem is how such unorganized structures can process signals in a precise and reliable way involving huge number of neurons. We will first propose a model for the computation based on general computation theory of convolution and correlation. Next we study the properties of this model and show that randomness is by far not a deficiency but a powerful feature enabling solution of the problems related to the precision of operation, noise and reliability of computational elements.

COMPUTATION OF CORRELATION/CONVOLUTION OF INPUT SIGNALS USING RANDOM MATRICES

Correlation and convolution are basic signal processing operations widely used because of many desirable properties and efficient calculation algorithms [3]. There have been many indications that these operations are also used in biological systems implemented by neurons [4], [5]. Here we will study how these operations could be implemented in neuronal systems taking into account their inherent randomness of operation. The cross-correlation of two sequences x_1 and x_2 of size N is defined as

$$r_{12}(j) = \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) x_2(n+j) \tag{1}$$

The convolution of two sequences x_1 and x_2 of size N is defined as

$$y(j) = \sum_{n=0}^{N-1} x_1(n) x_2(j-n) \tag{2}$$

Classical problem of signal processing is efficient computation of those operations. A well known solution is by the use of discrete and fast Fourier transform but this problem has been intensively culminating in the general theory by Winograd [6]. The computation of convolution and correlation can be formulated in general as

$$\mathbf{w} = C \cdot [(A \cdot \mathbf{u}) \otimes (B \cdot \mathbf{v})] \tag{3}$$

where the $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are input and output vectors respectively, A, B are transformation matrices, C is an "inverse" transformation matrix and $\mathbf{w}$

is the resulting convolution or correlation vector while $\otimes$ denotes point-to-point vector multiplication. The use of transformation matrices allows for the design of efficient computational algorithms. It is known that if matrices A, B and C are square of size $n \times n$ then the Fourier transform is unique transform satisfying (1). However, matrices A, B and C can also be rectangular and then there is considerably more freedom for the algorithm design. For digital systems, algorithms with low number of additions and multiplications have been derived especially for the case of short input vectors [6].

In this paper we will investigate the case which is inspired by the apparent structure of neuronal systems, that is when matrices A, B, and C have essentially random character, e.g. their columns are made of random uncorrelated sequences. First question is what is the set of minimal structural conditions which one has to impose on such matrices in order to get correct calculation of correlation/convolution vectors?

Let A and B will be two $n \times m$ rectangular matrices with column coefficients made of uniformly distributed random sequences. Such matrices will posess pseudoinverse which will be $A^{-1} = A^T$. This results from the fact that column sequences will be uncorrelated. Let A and B have the following form

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{22} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} ; B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{22} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

in our case m will be large $m >> n$.

When matrices A and B are used in (1) it is obvious that the "inverse transform" matrix C has to be of the size $m \times n$ and certain conditions have to be imposed on its structure in order to perform the correlation or convolution operations. Let the matrix C be defined as

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & \cdots & c_{nm} \end{bmatrix}$$

Then the output vector component w_t of (3) can be written as

$$w_t = \sum_{s=0}^{M-1} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} c_{ts} a_{ik} b_{il} u_k v_l \tag{4}$$

Now let's assume that the coeffcients of the matrix C are

$$c_{t,s} = a_{s(int[s/N])} b_{s(t-int[s/N])} \tag{5}$$

where $int[s/N]$ denotes integer part of s/N and $0 \leq s \leq M-1, M >> N$.

Substituting this into (4) one can see that the sequence of c_{ts} as defined is correlated with the sequence of product coefficients $a_{ik}b_{il}$ when $i = s, k = int[s/N], l = t - int[s/N]$ and decorrelated otherwise. For the case of correlation one obtains coefficients of convolution, for the case of decorrelation the sum of products is zero since sequences of coefficients are assumed random.

This model of calculating correlation and convolution by using correlation properties of random sequences is reminiscent of communication systems based on correlation e.g. CDMA detection. Our model shows that indeed signal transformation matrices can have random character and still the correct result can be obtained when using properly structured inverse matrix. After the transformation of input signals, there is rich "potential" information embedded in the transformation vectors. The required output result is obtained due to the structure of the inverse matrix.

One can also observe that computations in this model are possible only when the number of rows of input matrices (equal to the number of columns in the inverse matrix) is much bigger than the number of columns. Thus, by necessity the computational system has to be large. This is inherent feature of the model and may rise questions about the computational complexity and precision of calculations required. We shall investigate these issues next.

TESTING THE MODEL PERFORMANCE

We made tests with the calculation of correlation of two sequences using the model above. The tests were realized using Matlab. The typical size of the vectors and matrix used during the tests were: input vector of size 16, and the transformation matrix with the number of rows varying between 1000 to 50000. The computations are clearly not fitting well into the computer paradigm since they are memory consuming. Since the model is based on the extraction of correlated sequences from noise there will be an impact of noise on the precision of results.

The quality of results is evaluated by comparing correlations of two sequences calculated directly and with the use of random. The input vectors used in the tests are identical random sequences on which the correlation is performed. As expected, the quality of the result increases with the size of the transformation matrices : the longer are the random sequences the better are the correlation and decorrelation properties. This indicates that randomness and huge size of computational structure go hand-in-hand in this model and are indispensable from each other. In other words, small-sized systems of this kind could not perform the required computations.

INFLUENCE OF VARIOUS FACTORS ON THE RESULTS OF THE CORRELATION PERFORMED

We will now investigate the operation of the structures based on random matrices in conditions similar to real neuronal networks, i.e. in the presence of noise due to signal contamination, imprecise and unreliable computations. Intuitively one can say that using random sequences and correlation properties reduces the impact of the noise since it is uncorrelated with the random sequences used for computation. This is confirmed below and some interesting properties are observed.

Quantization

In basic model we made no assumptions about the way artithmetic operations are done. Here we investigate realistic case when operations are made with limited precision by quantizing calculations used in the computation of convolution and correlation. To measure the influence of quantization we made numerical experiments investigating different levels and quantization and dependence of the results on the size of random matrices. We could then plot the precision of output result as a function of the quantization step. The quantization step value is used to round every calculation step in the computation. The quantization step we used is centered on level zero, which means that for example a quantization step of value 1 will set to 0 the values inside $[-0.5; 0.5]$, to 1 the values inside $]0.5; 1.5]$ etc. (see Figure 1). Since random matrices with different sizes are used, the random sequences are normalized and the same number of quantization levels is used for calculations with those matrices.

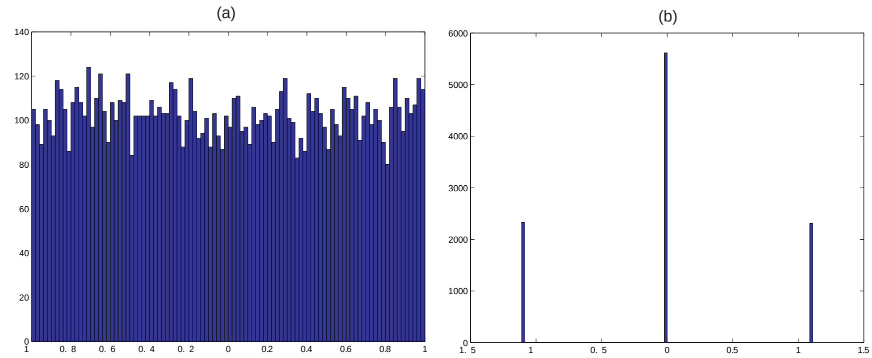

Figure 1: (a) : Histogram of the values of the uniform random transform matrix before quantization. As expected, the values are uniformly distributed. (b) : Histograms of the values of the same matrix after quantization with a (centered) quantize step of 1.1. We can see that the resulting matrix contains a lot of zeros in case of hard quantization.

Numerical results from experiments are shown in Figure 2.

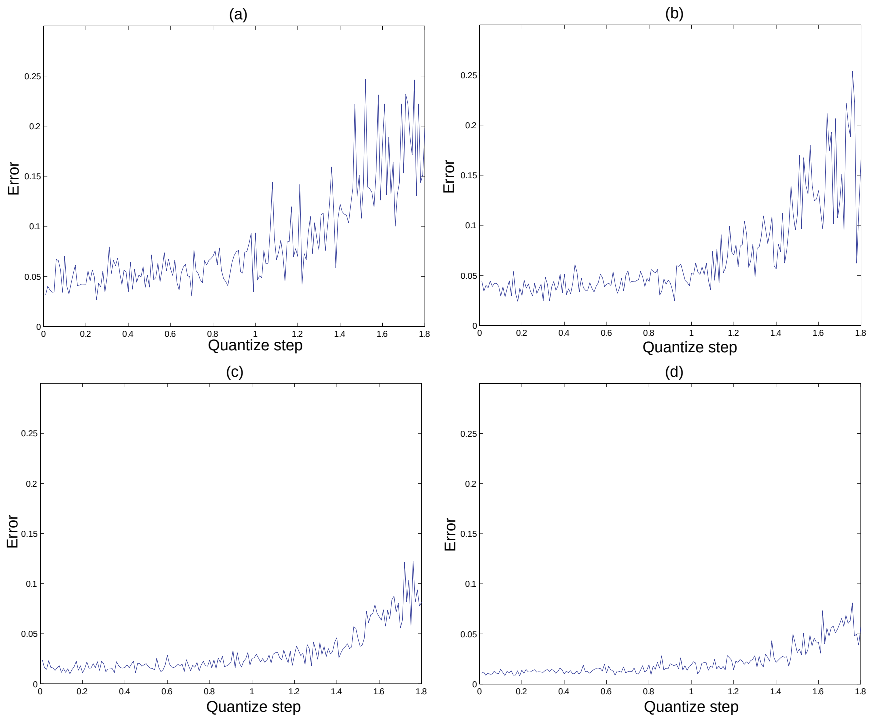

Figure 2: Average error between the original correlation vector and the one computed with our method with a (centered) quantize step varying from 0.01 to 1.8. The size of the matrix is m=500 (a), 1000 (b), 5000 (c) and 10000 (d). The average error between the original vector r and the vector r_q computed for a quantization q, is defined the following way : $e(q) = 1/N \sum_{i=1}^{N} |r(i) - r_q(i)|$ where r and r_q normalized and of size N.

From Figure 2, we observe that the quality of result depends mainly on the size of the matrix. The bigger is the matrix, the better is the result. Moreover, it is interesting to notice how the computation is robust even to hard quantization. For quantization step values bigger than 1, the components of the matrices are coded on 3 values only! For example for a precision of 1, the coefficients in the matrices are -1, 0 or 1. But the quality of the result is almost the same as the one with no quantization, if the matrix size is big enough (Figure 3). This shows that we can obtain good correlation results using only a 3-levels quantization for every component of the calculation at the exception of course of the input and output vectors.

Unreliable computations

It is well known that neuronal systems, especially synapses, are unreliable and operate in stochastic manner. We can model this by setting a percentage of the coeffcients of random matrices and intermediate vectors to zero.

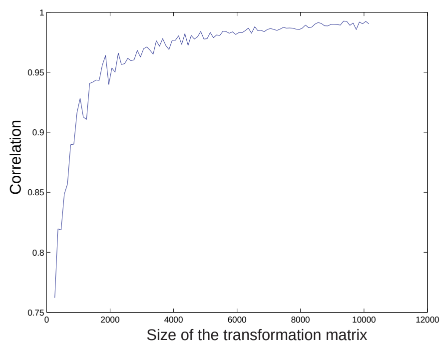

Figure 3: Correlation between the theoretical correlation vector and the one computed with our method in function of the size of the transformation matrix. In this case, the matrices elements are quantized to 3 values. The correlation represented on the vertical axis is normalized therefore the value one means that the two sequences are fully correlated (i.e. they are identical if normalized) while the value zero means that they are totally uncorrelated.

The Figure 4 shows the relation between the percentage of zeros set and the accuracy of the results for matrices of size 1000 to 10000. We can see that the accuracy of the output result is strongly related to the matrix size but for sizes above 5000 30% of coefficients can be removed with very little imapct on the precision of output result.

INTERPRETATION AS A NEURONAL NETWORK MODEL

We can consider our approach to calculation of convolution and correlation using random matrices as a model of operation of neuronal network with synaptic weights represented by the coefficients of the matrix. Single row of the matrix represents synaptic weights for one neuron. Transformation of input vectors by the transformation matrices A and B is an initial step in the calculation but does not produce yet any "useful" information. This is done with the use of the inverse matrix structure. Basic calculations like products of vector coefficients are done by using correlation properties of random sequences. This allows to extract useful information from background of noise, principle used also in CDMA communication systems. The model has huge structural redundancy but this makes it robust for noise and unreliable operation. One can also notice that the design complexity is low since coefficients in the algorithms are obtained from random generator. Besides these properties the model naturally incorporates quantization and thresholding in calculations. Even hard quantization of matrix coefficients to just several levels gives proper calculation results.

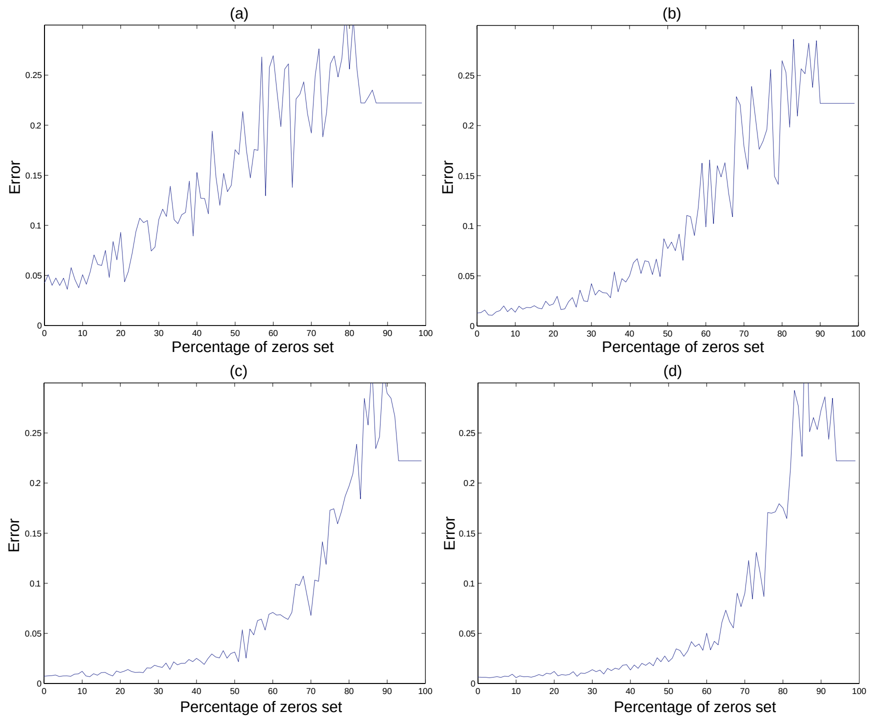

Figure 4: Average error between the original correlation vector and the one computed with our method in function of the percentage of values set to zero in the matrix. The size of the matrix is (a) 1000, (b) 10000, (c) 50000 and (d) 100000. The average error between the theoretical vector r and the vector r_z computed for a percentage of zeros z, is defined the following way : $e(z) = 1/N \sum_{i=1}^{N} |r(i) - r_z(i)|$ where r and r_z normalized and of size N. The matrices elements are still quantized on 3 values as in the previous section. We observe that with a matrix size of 100000, the reliability is still quite good even with 50% of the values set to zero.

DECORRELATION OF PATTERNS USING RANDOM COEFFICIENT MATRICES

Our basic model is strictly linear. However, with quantization and thresholding nonlinearities are introduced. Nonlinearities are necessary and commonly used in neural networks to increase separability of vectors.

To investigate if this is also the case in our model, we transformed input vectors with random matrix with quantization and thresholding. We obtained transformed vectors of much larger size and checked their correlation properties. We used random matrices quantized to 3 levels (-1 0 and 1) with row size typically in the order of 100000. Input vectors are multiplied by these matrices and resulting output vectors are thresholded to 3 levels by the threshold function from Figure 5.

The interest is in the decorrelation properties of such processing. For testing this, input vectors of size 16 were created and their correlation measured. Then the correlation of transformed vectors is measured and compared.

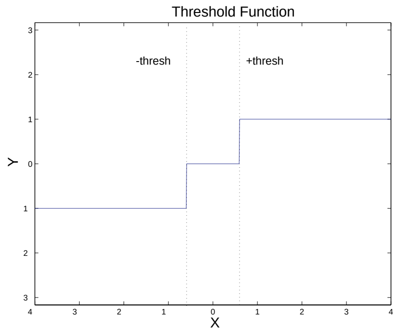

Figure 5: Threshold function used to "quantize" data on 3 levels. The values are set to zero if they are inside $[-thresh; thresh]$ otherwise they are set to -1 or 1.

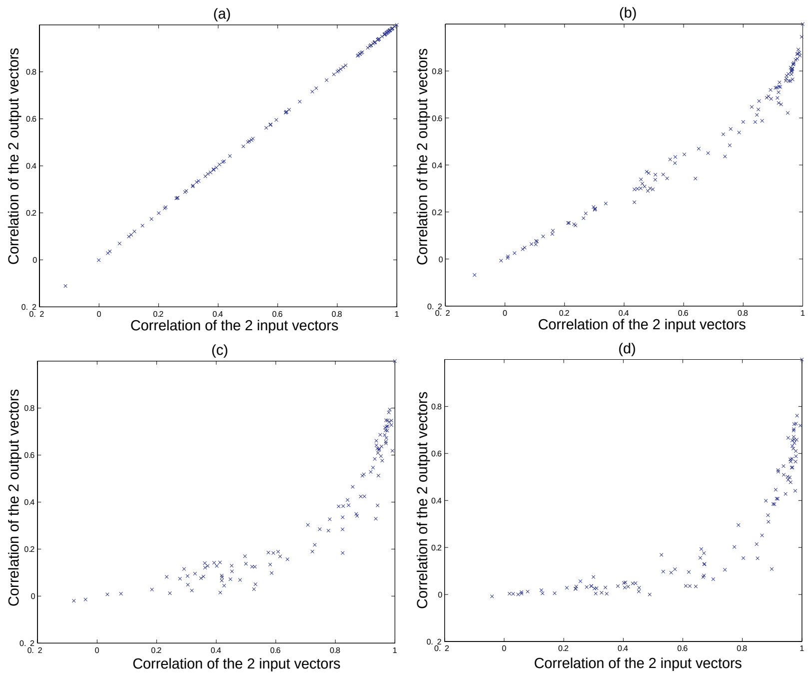

Figure 6: Correlation of the 2 expanded vectors in function of the correlation of the 2 inputs vectors. The transformation matrix is of height 500000. On (a) the thresholding step has been ommited. On (b), (c) and (d), the threshold value is respectively 2, 4 and 5. A bigger threshold value does not allow the information to be preserved for a matrix size of 500000.

We can see in Figure 6 (a) that when there is no thresholding, the operation is linear and there is no decorrelation. In Figure 6 (b), (c) and (d), we can see the influence of the threshold value on the decorrelation property. In (d), we observe that if the correlation of the input vectors is smaller than 0.8, the correlation of the expanded vectors will generally be smaller than 0.3. We can observe that if the two input vectors are identical, the decorrelation does not happen. But if vectors are different, the decorrelation effect will take place.

CONCLUSION

In this paper we investigated a model for the computation and convolution based on random matrices. The model has many features which are reflecting the structure of biological neuronal systems. It is shown that structural complexity is inevitable necessity in this model. But this complexity provides many benefits like robustness to noise and unreliable operation, low precision computations and decorrelation properties. Computations utilizing correlation properties of random sequences are used most notably in CDMA communication systems with striking advantages and here we show that similar principles might be at work in biological neuronal systems.

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